Multivariable Calculus

Quiz 5 **SOLUTIONS**

1) Consider the curve that is the intersection of the cylinder $x^2 + y^2 = 4$ with the surface $z = xy$.

a) Find a parametrization of this curve in the form $\vec{r}(\theta) = \langle x(\theta),\ y(\theta),\ z(\theta)\rangle$.
HINT: x and y lie on a circle of radius 2! That should give an obvious way to parameterize those coordinates!

Solution:

$$\begin{aligned}\vec{r}(\theta) &= \langle 2\cos(\theta),\ 2\sin(\theta),\ 4\sin(\theta)\cos(\theta)\rangle \\ &= \langle 2\cos(\theta),\ 2\sin(\theta),\ 2\sin(2\theta)\rangle\end{aligned}$$

b) Compute the derivative of this parametrization, $\vec{r}\,'(\theta)$.

Solution:

$$\vec{r}\,'(\theta) = \langle -2\sin(\theta),\ 2\cos(\theta),\ 4\cos(2\theta)\rangle$$

c) Show that the speed of this parametrization is given by

$$\|\vec{r}\,'(\theta)\| = 2\sqrt{1 + 4\cos^2(2\theta)}.$$

Solution:

$$\begin{aligned}\|\vec{r}\,'(\theta)\| &= \|\langle -2\sin(\theta),\ 2\cos(\theta),\ 4\cos(2\theta)\rangle\| \\ &= \sqrt{4\sin^2(\theta) + 4\cos^2(\theta) + 16\cos^2(2\theta)} \\ &= \sqrt{4 + 16\cos^2(2\theta)} \\ &= 2\sqrt{1 + 4\cos^2(2\theta)}\end{aligned}$$

TURN OVER

2) Find the arc length of the space curve $\vec{r}(t) = \langle t^2,\ 2t,\ \ln(t)\rangle$ on $1 \le t \le e$.
HINT: $\|\vec{r}\,'(t)\|^2$ should be a perfect square!

Solution:

$$\begin{aligned}
L &= \int_1^e \|\vec{r}\,'(t)\|\,dt \\
&= \int_1^e \left\|\left\langle 2t, 2, \frac{1}{t}\right\rangle\right\|\,dt \\
&= \int_1^e \sqrt{4t^2 + 4 + \frac{1}{t^2}}\,dt \\
&= \int_1^e \sqrt{\left(2t + \frac{1}{t}\right)^2}\,dt \\
&= \int_1^e \left(2t + \frac{1}{t}\right)\,dt \\
&= \left[t^2 + \ln|t|\right]_1^e \\
&= (e^2 + 1) - 1 = e^2
\end{aligned}$$